

# Measuring Sovereign Risk With Contingent Claims Analysis: The Empirical Evidence in Southeast Asia Credit Markets

**Ho Hong Hai**

*Foreign Trade University, Vietnam*

*Email: hai.ho@ftu.edu.vn*

**Tran Duy Long**

*K&G Vietnam Investments JSC, Vietnam*

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## **Abstract**

*This paper focuses on examining the degree to which the Contingent Claims Analysis is useful for Southeast Asia markets. Such a framework is initially developed for analyzing corporate sector default based on the theory of Black-Scholes options pricing and the structure of accounting balance sheet, and then adapted to the sovereign balance sheet in a way that can help forecast credit spreads and evaluate the impacts of risk transferred from other sectors. Robustness checks indicate that sovereign CCA is consistent with most markets in the sample. Scenario analysis interprets two prospects with assumptions on economic growth and capital structure of the Vietnam government in the short-term future.*

**Keywords:** Capital structure; contingent pricing; sovereign distress.

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## 1. Introduction

In the era of cross-border cooperation, the global economy has become significantly more vulnerable due to the uncertainty of capital flows in financial markets, leading to the issues of low liquidity and enormous credit risk. The empirical evidence denotes increasingly severe damage from financial crises over the last few decades, such as the Asian financial crisis (1997), the Global financial crisis (2008), the European sovereign debt crisis (2009), and the most recent Chinese stock market crash (2015). What we can observe from these events is that risk is transferred among different sectors and spread out over time. And when the crises hits emerging markets, the consequence is more devastating due to snowball effects. The vast Southeast Asian currencies, for instance, plunged last summer and hit a trough in the prior 10-year period, which conjured up memories of the 1997 Asian financial crisis. Since international collaboration has been the main trend in contemporary financial investments, the worldwide aggregate market is getting more sensitive and sophisticated. Therefore, a comprehensive framework to identify and manage risk is necessary to meet the need for analyzing and hopefully preventing large-scale recession and financial distress.

Basically, an initial outlay for a financial project of any type is expected to bring back a positive net profit in the future. This expectation raises a fundamental question on the relation between risk and return. The Capital Assets Pricing Model (CAPM) introduced the first coherent theoretical structure to answer the question. In particular, this model devises a unique indicator implying the exposure to

systematic risk for a certain investment that is called the “risk premium”. An excess required return for an unstable financial market easily makes sense, but the differences among countries requires more. Comparing the United States’ developed market and the Southeast Asian emerging markets, we plainly recognize the need of considering another premium for risk at a macro economy scale. The adjusted CAPM also takes the country risk into account and applies an additional risk premium when it comes to those markets.

There are plenty of theories that discuss how country risk premium could be calculated. Sovereign bond default spreads, relative equity market standard deviations, or the combination of the two approaches all help determine which number is most consistent with a certain situation. However, these measures result in no conclusion on root causes for fluctuations in the premium. It is meaningless if we figure out some errors but have no idea about how to fix them. In contrast, the case that potential outcome is a bit obscure, yet we know precisely where it stems from, seems to be much more accessible. In addition, the core function of a useful risk measure is to identify and manage catastrophe prior to its actual outbreak. Eventually, there are three objectives for effective risk analysis. First, it must identify existing mismatches in financial data. Second, it should connect uncertainty inherent in the data that can affect assets’ value relative to promised payments on debt obligations and ultimately drive default risk. Third, risk exposure must be denoted under quantifiable indicators, revealing whether the default risk is building or subsiding.

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Accordingly, an effective approach that meets all three objectives has been widely applied since the 1997 Asian crisis to evaluate the risk posed by probably vulnerable components in sectoral balance sheets, including in the corporate, financial, and public sectors. Such an approach accesses the nonpayment risk of debt in an entity by means of its own debt structure, and considers the equity and contractual liabilities as the contingent claims on the assets, so it is called contingent claim analysis (CCA). The model in detail uses the basic equation of a balance sheet along with market prices and volatility to result in simple risk indicators that might illustrate forward looking events, which means it provides a marked-to-market balance sheet instead of a book-valued balance sheet as usual.

The first milestone of the CCA framework was placed by Black and Scholes with their publication on pricing of options and corporate liabilities in 1973 (Black and Scholes, 1973). In their paper, they stated at the beginning that almost all corporate liabilities can be viewed as combinations of options; therefore the option formula and analysis are applicable to corporate liabilities such as common stock and corporate bonds. The most valuable contribution of the research was a huge mathematical determination where option price is calculated by functions of stock price and timing variables with observable or readily estimated factors. At the same time, Robert Merton, Assistant Professor of Finance in the Massachusetts Institute of Technology, also published his paper entitled "Theory of Rational Option Pricing" (Merton, 1973), discussing extension of the option theory to the pricing of corporate liabilities.

Four years later, Merton introduced the first application of the discussed extension, explaining his attempts to measure the risk exposure in the financial sector. He considered including bank loans, as a part of a financial package, a guarantee by a third party and derived formula to evaluate the cost on the guarantor. The idea behind the determination is the identity between loan guarantees and common stock put option, which he called "an isomorphic correspondence". The model revolved around a bullet debt of a firm and the impact of a third-party guarantee to the debt. Merton then supposed the firm was a bank and the debt issue corresponded to deposits, and finally, he resulted in the estimated cost of deposit insurance under a variety of deposit-to-asset target ratios and volatility of the assets. However, there was no empirical examination for such a framework until Chan-Lau et al. (2004) published their statistical tests and forecast on bank vulnerabilities in emerging markets. Their data availability constrained the study to 38 banks from fourteen different emerging market countries, such as Thailand, Hong Kong, Brazil and Argentina, during the 8-year period from 1997 to 2003. The group of researchers found that it is able to forecast bank distress, which was defined in their study as a rating downgrade to CCC or below, up to 9 months ahead in-sample. They also used logit and prohibit regression models to construct default probability as an understandable measure of financial difficulty. The next step was taken by Gapen et al. (2005) with their application of CCA at a sovereign level. The examination was conducted in 12 emerging market economies by using robustness checks, regression and scenario analysis on the CCA risk indicators. The

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results showed the risk indicators to be robust and significant compared to market observed variables, which were not used as inputs. Another study by Gray et al. (2007) also pointed out similar conclusions as the CCA model was applied to Brazil in the volatile period of 2002-2005. The long-standing development of the CCA framework seems to be ongoing or at least in this paper. It was applied more broadly for governments in Vietnam and other South-east Asia countries almost a decade later.

This paper uses the methodology following Gapen et al. (2005) and represents the very first step using the modern theory to approach the financial position of the authorities in the mentioned emerging markets.

A set of key credit risk indicators introduced in this paper includes: distance-to-distress, probability of default, credit spread, and the market value of risky foreign currency denominated debt. To determine the usefulness of the credit risk indicators as a collective barometer of sovereign risk, they are subjected to a robustness test using observed market data for a sample of emerging market countries. Since Sovereign bond spreads were not put into the model as inputs, a high correlation between the data on spreads and the derived risk indicators would suggest that the indicators can be confidently used as reasonable measures of sovereign credit risk. The robustness checks are applied in the individual cases of four South-east Asian countries, including Indonesia, Malaysia, the Philippines, and Vietnam, and examined across all the countries. In addition, a detailed analysis is conducted with data from the particular case of Vietnam.

## **2. Theoretical framework and methodologies**

### **2.1. Theoretical basis and analysis framework**

#### *2.1.1. An initial rational framework*

The starting point of contingent claim analysis derived from Merton's (1973) model is accessing the solvency of debt issuers. Consider a firm that issues bonds or borrows funds at a given time with a certain maturity. The question arising is whether the firm has enough assets to honor its commitments. In simple terms, the firm will be not able to fulfill its obligations if the payment surpasses, at maturity, its assets value, which in turn leads to a declaration of bankruptcy. Apparently, deciding on making the payment at debt maturity is very similar to a process of exercising a call option. Recall that a call option gives the holder the right, but not the obligation, to buy an underlying asset at a specified price (*strike price*), at a predetermined point in time<sup>2</sup>. In this arrangement, the holder will buy the asset if its market value exceeds the strike price, or otherwise the option will not be exercised. To clarify the similarities between these two decisions, having a simple model without frictions, the firm will pay its liabilities if the asset value exceeds its nominal debt. Intuitively, paying debt can be seen as repurchasing the assets, because the assets are funded by loans, which lawfully means that unless the firm fulfills its obligations, the ownership of the assets will belong to its debt holders. Theoretically, value of nominal debt has similar features with the exercise price. Enterprise value plays the role of the value of the underlying asset, and payment due date can be considered as maturity of the option. Specifically, a lender gives a firm's owner an amount of loan, and the right to decide on whether or not to pay back the debt. Subsequently, the firm

turns the loan into assets serving its business. For this reason, the assets are equivalent to the underlying asset in a call option.

As debt payments are contingent on asset value, there is no certainty of their payments. Therefore, using a sole discount factor based on a risk-free rate, as in most existing debt-pricing models, is never enough to precisely evaluate the market value of debt. Analysis should take uncertainty into account. Once again this directs us toward option pricing theory, but the destination is put option this time. A put option gives the holder the right, but not obligation, to sell the underlying asset at a predetermined price at option maturity. For debt holders, the present value of riskless debt (*default-free debt*) should be equal to the value of risky debt plus a guarantee on that debt. In case of non-repayment, the creditors reserve the right to partially collect the debt by liquidating the debtor's assets. Hence, the debt guarantee can be achieved through a put option, which will be exercised if the value of debt – exercise price – is higher than total assets of the firm – the underlying asset.

### 2.1.2. Formulated contingent claims approach

#### 2.1.2.1 Merton model equations for pricing contingent claims

The essential symmetry of a balance sheet is shown in the accounting equation. However, when it comes to Merton's model, the equation was slightly adjusted to represent that the total market value of assets at time  $t$  equals the sum of all contingent claims on the assets, including equity and risky debt maturing at time  $T$ .

$$\text{Assets} = \text{Equity} + \text{Risky Debt}$$

$$A(t) = E(t) + D(t) \quad (1)$$

The value of assets follows Winner's process and probably drops below the debt payments. Equity can be modeled in an implicit call option on the assets, with an exercise price equal to the promised payments,  $B$ , that matures in  $T-t$  periods. Risky debt can be calculated as the difference between default-free debt and debt guarantee. The guarantee is equivalent in value to a put option under the same conditions with the call.

Risky Debt = Default - free Debt - Debt guarantees

$$D(t) = Be^{-r(T-t)} - P(t) \quad (2)$$

And Equity = Call option on Assets  
(using the Black-Scholes formula)

$$E(t) = AN(d_1) - Be^{-rT}N(d_2) \quad (3)$$

$$d_1 = \frac{\ln\left(\frac{A}{B}\right) + \left(r + \frac{\sigma_A^2}{2}\right)(T-t)}{\sigma_A \sqrt{(T-t)}}$$

$$d_2 = d_1 - \sigma_A \sqrt{(T-t)}$$

Equation (3) has two unknowns,  $A$  and  $\sigma_A$ . In order to obtain their value, it is necessary to impose a second condition. One possibility is a statement that equity,  $E$ , also follows a generalized Winner process but with different parameters from  $A$ . Applying Wiener process' definition and equating the volatility terms, we obtain

$$E\sigma_E = A\sigma_A N(d_1) \quad (4), \text{ where } \sigma_E \text{ is volatility of equity returns.}$$

Solving the system composed equations (3) and (4) for each moment is possible to obtain a time series value for  $A$  and  $\sigma_A$ . To achieve this challenge, this paper applied the Bivariate Newton Root-Finding Method, which uses it-

eration, producing a sequence of numbers that converge towards a limit. The first values of this series (initial guesses) are the estimates of asset value,  $A_t = E_t + B_t$ , and asset volatility,  $\sigma_A = \frac{\sigma_E \times E_t}{E_t + B_t}$ . The method computes subsequent values by evaluating an auxiliary function on the preceding value. The computation stops when it reaches the limit of 20 iterations or the numerical error between two consecutive results is less than  $10e-10$ .

### 2.1.2.2. Calculations on credit risk indicators

The CCA results in a series of measures used in risk analysis. First of all, distance-to-distress,  $d_2$ , yields the number of standard deviations the asset value is from the distress barrier. Lower market value of assets, higher levels of nominal debt, and higher levels of asset volatility all serve to decrease this indicator. Distance-to-distress for a hypothetical asset return process is illustrated in Figure 1.

In formula representation,

distance - to - distress =  $d_2 =$

$$\frac{\ln(A_t / B_t) + (r - \sigma_A^2 / 2)(T - t)}{\sigma_A \sqrt{(T - t)}} \quad (5)$$

The call put parity in option pricing theory also yields a measure of probability of default, commonly referred to as the risk-neutral default probability which is the area below the distress barrier as shown in Figure 1.<sup>3</sup> Thus, the risk-neutral default probability (RNDP) is,

$$\text{RNDP} = N(-d_2) \quad (6),$$

where  $N(-d_2)$  is the cumulative normal distribution at the distance-to-distress,  $d_2$ .

The other two useful sovereign risk indica-

tors that can be obtained using the CCA are the credit risk premium, and the market value of risky senior debt. The equation (2) defines the value of risky debt which can be expressed as

$$D(t) = Be^{-r(T-t)} - [Be^{-r(T-t)}N(-d_2) - V_A N(-d_1)] \quad (7)$$

As the ratio of assets to the default-free value of debt rises or the asset volatility declines, the value of risky debt increases, and vice versa. In other words, if a firm becomes wealthier and its income flows less uncertain, the market value of its debt will become more valuable. Manipulating equation (6) results in an estimate of the risk-neutral credit spread (RNS) of,

$$\text{RNS} = y - r_f = \frac{-\ln(D(t)/B(t))}{T - t} - r_f \quad (8)$$

The right-hand side of equation (8) represents the yield to maturity on risky debt less the risk-free rate of interest and is therefore equivalent to a credit risk premium. Intuitively, both increases in the ratio of assets to nominal debt and decreases in the asset volatility reduce the risk premium, which implies that the firm has a thicker cushion of assets to self-protect from negative shocks and the cushion is more stable.

### 2.1.3. CCA in sovereign credit risk analysis

#### 2.1.3.1. Transferring accounting balance sheet from corporate to public sector

The way a sovereign manages its capital structure can be considered as corporate operation. There are sufficient similarities between individual firm risk and sovereign risk to suggest a reasonable transfer of the contingent claims approach from corporate to sovereign risk analysis. The majority of a firm's assets include cash and the present value of potential profit (stream of revenues minus expenditures).

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Other assets such as fixed assets, tools, equipment, and inventories, which are recognized as costs along with the stream of income, should be abandoned to avoid overlapping. On the liabilities side, a firm has senior debt, subordinated debt, and equity. Market capitalization of the firm is equal to the price of equity multiplied by the number of shares issued. Turning to a sovereign balance sheet, main assets consist of international reserves and present value of the net fiscal surplus (stream of revenues minus expenditures). Analogous to firms, a sovereign also has land or other assets which are not included in the definition of its assets. Sovereign liabilities comprise foreign currency debt. The sovereign also has local currency debt and base money, which yields the foreign currency value of domestic currency liabilities when multiplied by the exchange rate.

Another similarity can be seen from analyzing post-default behaviors of firms and sovereigns. Corporate sector defaults commonly trigger a bankruptcy process which is well defined in most countries whereby creditors are assigned their claim to a firm's assets based on the legally specified seniority of liabilities in the capital structure. As debt is senior to equity in the event of default, bondholders may choose to liquidate remaining assets to recover a cash payment in some cases, or replace the board of directors and receive new junior claims (equity) in the others. Seniority in sovereign liabilities is not defined through legal status as in the corporate sector, but may be inferred from examining the behavior of governments during distress. Thus, governments in periods of stress tend to attempt to maintain their existing foreign-currency debt status and turn such senior

debt into domestic-currency liabilities.<sup>4</sup> The payment of sovereign senior debt requires the acquisition of foreign currency that is limited in the revenue of a government. In comparison, a government has much more flexibility to issue, repurchase, and restructure the local-currency debt, which has certain "equity-like features". Therefore, governments sometimes execute capital restriction to prevent convertibility and preserve remaining international reserves for their external debt obligations, but when the restriction turns out to be insufficient, governments have insisted on mandatory turnover or restructuring of domestic-currency debt without adding other foreign-currency credits to their liabilities.

#### *2.1.3.2. Structure of sovereign accounting balance sheet*

The contingent claims sovereign balance sheet is constructed from the basic accounting balance sheet of the government and monetary authorities. Figure 5 demonstrates the consolidated version of the sovereign balance sheet. Note that as a segregated entity, the government has assets including the claim on a portion of international reserves held by the monetary authorities, which also appears in the balance sheet of the monetary authorities as a liability item. Hence, the two entries are offset against each other once they enter the consolidated sovereign balance sheet. Similarly, the obligations owed by the government to the monetary authorities are offset by the credit to the government on the assets side of the monetary authority balance sheet. Eventually, the sovereign balance sheet can be broken down as:

Assets include:

*Foreign reserves* – Net international reserves

**Figure 1: Sovereign consolidated balance sheet**

<b>ASSETS</b>	<b>LIABILITIES</b>
<p><b>Foreign Reserves</b></p> <p><b>Local-currency Assets (in Foreign-currency Terms)</b> [=Net fiscal assets – Guarantees + Other assets]</p> <p><u>Claim on portion of foreign reserves</u> (Government Asset)</p> <p><u>Credit to Government</u> (Monetary Authority Asset)</p>	<p><b>Local-currency Liabilities (in Foreign-currency Terms)</b> [=Local-currency debt + Base money]</p> <p><b>Foreign-currency Debt</b></p> <p><u>Claim on portion of foreign reserves</u> (Monetary Authority Liability)</p> <p><u>Credit from Monetary Authority</u> (Government Liability)</p>

of the public sector.

*Net Fiscal Asset* – Items related to revenues, taxes, and expenditures. Subtracting the present value of non-discretionary expenditures, that the government has to maintain prior to giving up paying the debt, from the present value of taxes and revenues yields the net fiscal asset which also is equal to the present value of the primary fiscal surplus over time (fiscal surplus minus interest payments).

*Other Assets* – Equity in public enterprises, value of the public sector’s monopoly, and other financial and non-financial assets.

Liabilities consist of

*Base money* – Currency in circulation, bank reserves (required bank reserves, excess reserves, vault cash).

*Local-currency debt* – Domestic debt of the government and monetary authorities, held

outside of the monetary authority and the government.

*Foreign-currency debt* – Sovereign debt denominated in foreign currency, held primarily by foreigners.

*Guarantees* – Implicit or explicit financial guarantees to “too-important-to-fail” entities (banks, monopoly enterprises or contingent pension/social obligations).<sup>5</sup>

#### 2.1.3.3. Constructing the CCA model for sovereign credit risk analysis

Similar to corporate sector default, sovereign distress is defined as the event of a decrease in the value of sovereign assets to or below the promised payment that the sovereign abides by. The amount of payment also makes a distress barrier, which in this paper, is equivalent in value to short-term debt plus one-half of long-term debt plus interest payments up to

time  $t$ .<sup>6</sup> A two-phase seniority of debt repayment is applied to estimate the implied asset value,  $V_{sov}$ , and the implied asset volatility,  $\sigma_{SSov}$ . Since foreign-currency debt in the sovereign balance sheet can be viewed as “senior claim” and local-currency liabilities as “junior claim” on sovereign assets, the two liability entries can be computed as contingent claims in the CCA model.<sup>7</sup> Thus, the risky foreign-currency debt is equivalent to the default-free value of foreign-currency debt minus an implicit put option. Sovereign local-currency liabilities,  $LCL_S$ , which are similar to corporate equity, can be computed as an implicit call option on sovereign assets,  $V_{SSov}$ , with an exercise price equal to the distress barrier,  $B_f$ .

$$LCL_S = V_{SSov} N(d_1) - B_f e^{-rT} N(d_2)$$

Based on the linkage of the sovereign balance sheet, we can also calculate the local-currency liabilities as

$$LCL_S = M + B_{ds,t=0} = \frac{(M_{LC} e^{r_d T} + B_d) e^{-rT}}{X_F}$$

and the volatility of local-currency liabilities as  $\sigma_{SSov} = f_2(M, B_d, r_d, \sigma_M, \sigma_{B_d}, \sigma_{X_F}, X_F, \rho_{B_d, X_F}, \rho_{M, B_d})$

where  $M_{LC}$  is base money in local-currency terms;  $r_d$  is domestic risk-free rate;  $B_d$  is local-currency debt,  $X_F$  is forward exchange rate;  $\sigma_M, \sigma_{B_d}, \sigma_{X_F}$  are volatility parameters of base money, local-currency debt, and forward exchange rate, respectively;  $\rho_{B_d, X_F}$  is correlation of local-currency debt and forward exchange rate;  $\rho_{M, B_d}$  is correlation of base money and local-currency debt.

The remaining calculations are similar to the contingent claim analysis process described in section 2.2 for a hypothetical firm.

## 2.2. Methodology

### 2.2.1. Robustness of sovereign credit risk indicators

The degree to which the contingent claim risk indicators closely parallel actual market data will indicate their usefulness as a leading indicator of sovereign risk. To this conclusion, a historical time series of risk indicators on a yearly frequency is compared to actual market data for four Southeast Asian countries, including Indonesia, Malaysia, the Philippines, and Vietnam. The market data used in this paper is the Sovereign Bond Spread which was obtained from the international bond market. Robustness of the indicators is examined through their correlation with actual data.

For the scope of this paper, the correlation between risk-neutral sovereign credit spread and sovereign bond spread is calculated using the Pearson’s correlation coefficient as such a calculation implicitly assumes linear relationships among normally distributed variables. In this case, they are both credit risk premiums. The Spearman’s rho correlation, on the other hand, is a less restrictive measure to gauge relationships among variables since it does not impose any linearity assumptions. Therefore, the relationships between distance-to-distress indicators and Sovereign bond spread are examined by the Spearman’s rho correlation.

### 2.2.2. Scenario analysis

The structural models calibrated using the contingent claims framework and unique to each economy can be used with scenario analysis to evaluate shocks and policies. The objective is to estimate the potential effects of changes in economic conditions and impact of government policies on sovereign credit risk

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and sensitivity indicators. To begin with, a baseline balance sheet for the Vietnam government in 2015 is calibrated and the resulting baseline risk indicators and sensitivity measures are reported. Scenario analysis is then conducted using predictions of the Vietnam economy for two years to come, 2016 and 2018, proclaimed by the World Bank. The resulting point estimates for the credit risk indicators and sensitivity measures are compared to the baseline set of indicators.

### **3. Empirical investigation of contingent claim analysis on measuring sovereign credit risk**

#### ***3.1. Robustness checks of sovereign credit risk indicators***

If the model output is robust, distance-to-distress must be negatively correlated with Sovereign bond spread. Once distance-to-distress increases, credit risk reduces, which is reflected in a lower Sovereign bond spread. Figure 2 represents the correlation of distance-to-distress indicators and Sovereign bond spread. Thus, a high correlation (R-squared is 0.75) and the negative exponential relationship can be seen from the chart given. Table 1 shows the correlations and their significance of Sovereign bond spread and distance-to-distress (Spearman's rho correlation), and of Sovereign bond spread and risk-neutral sovereign credit spread (Pearson correlation). All the figures indicate high correlations in both of the two pairs examined. In addition, almost all of the sig. values fall below the significance levels, except in the case of the Spearman's correlation for Malaysia. This is probably due to the limitation of market data resources which could not be observed in a unique source, leading to the discrepancies

among the measurements. Also, the assumption of a risk-neutral world could not be compatible with this market.

As a second check on robustness, the risk-neutral probability of default for each country is compared to the Sovereign bond spread. Figure 3 displays the expected positive relationship between the two variables. The correlation of the risk-neutral probability of default and Sovereign bond spread is similar to the correlation of the risk-neutral sovereign credit spread and Sovereign bond spread, which is reported in Table 1, whereby the figures all indicate strong positive correlations (values of 0.7 to 1.0) and high degrees of significance (sig. values below 0.01).

#### ***3.2. Application of the CCA model to the analysis of Vietnam sovereign credit risk in the period 2001-2014***

In the early 2000s, the gap between the distress barrier and sovereign assets value for Vietnam as shown in Figure 4 is extremely narrow, yet the probability of default merely remained around a trough of 4%. That situation resulted from a low volatility of sovereign assets (around 15%), and the greater value of local-currency liabilities compared to the respective distress barrier. The movements on the actual market also showed a 5-year period during which Vietnam economy remained stable with a high GDP growth rate of 7% and great development potentials.

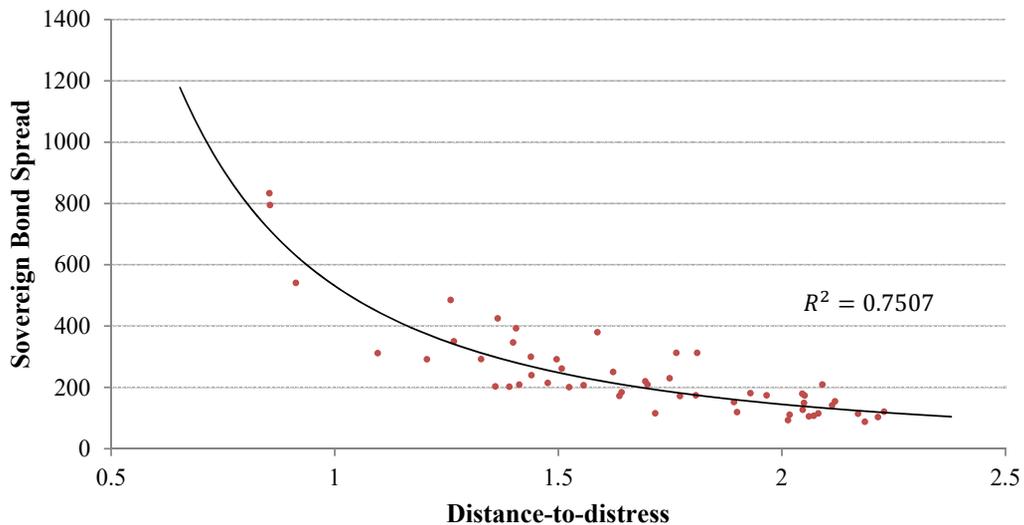
The probability of default had taken up since 2007, reaching a peak of 19.6% in 2008 and nearly approaching this degree in 2011. These indicators were matching with the actual events during the same period, whereby the effect from the global financial crisis engulfed the

**Figure 2: Distance-to-distress and Sovereign bond spread**

Distance-to-distress,  $d_2$ , is expressed in Black-Scholes formula as,  $d_2 = \frac{\ln(A_t/B_t) + (r_f - \sigma_A^2/2)t}{\sigma_A \sqrt{t}}$ . Sovereign bond spread falls to the group of credit risk premium along with CCA risk-neutral credit spread,  $RNS = y - r_f$ , where  $y = \frac{\ln(B_t/D_t)}{t}$ .

Combining with the definition of risky debt,  $D_t = Be^{-r_f t} - P_t$ , we obtain  $RNS = -\frac{1}{t} \ln \left( N(d_2) + \frac{AN(d_1)}{Be^{-r_f t}} \right)$ . Therefore, Sovereign bond spread, that is similar to risk-neutral credit spread, has a negative exponential correlation with distance-to-distress,  $d_2$ .

The coefficient of determination is  $R^2 = 0.7507$ .

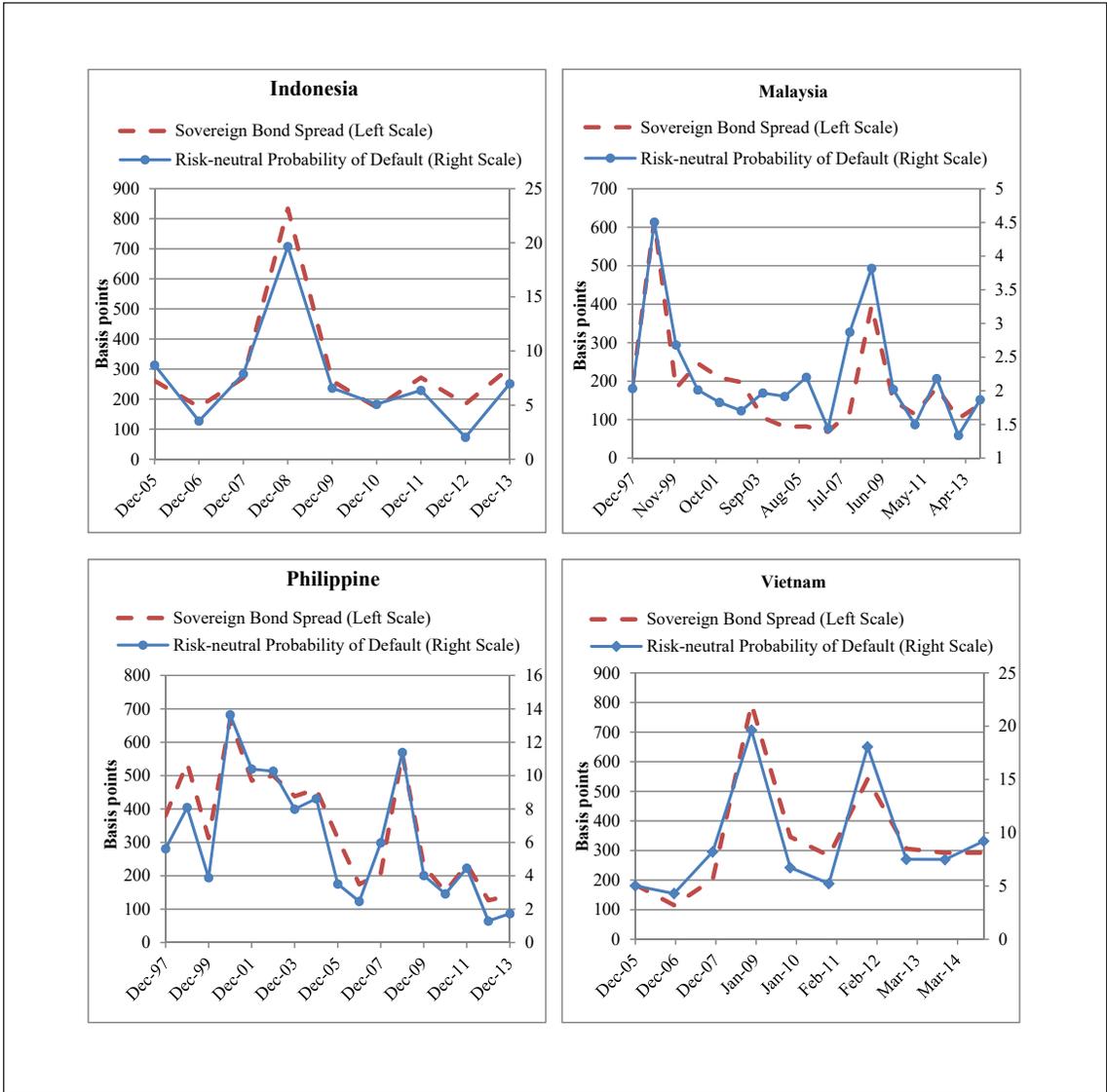


Vietnamese economy in a downward growth spiral with low GDP growth rates over the following few years. Especially in 2011, the authorities were confronted with a wide range of negative shocks toward the economy, such as the 18% inflation, insufficient investments from the public sector, a frozen housing estate market and around 50000 incidents of corporate default or bankruptcy. As an expert's per-

spective, Standard & Poors also downgraded Vietnam credit ratings from BB to BB-.

The CCA outputs for Vietnam's economy at the end of 2014 anticipate a volatile short-term prospect. The rapidly increasing base money and vulnerable exchange rate both trigger a high volatility of sovereign assets which in turn brings the government closer to distressed status. However, one positive signal here is

**Figure 3: Risk-neutral probability of default compared to sovereign bond spread**



that the gap between the distress barrier versus the sovereign asset value have been widening over time, which means the credit risk has been transferred to non-public sectors, keeping the government safe on its external obligations.

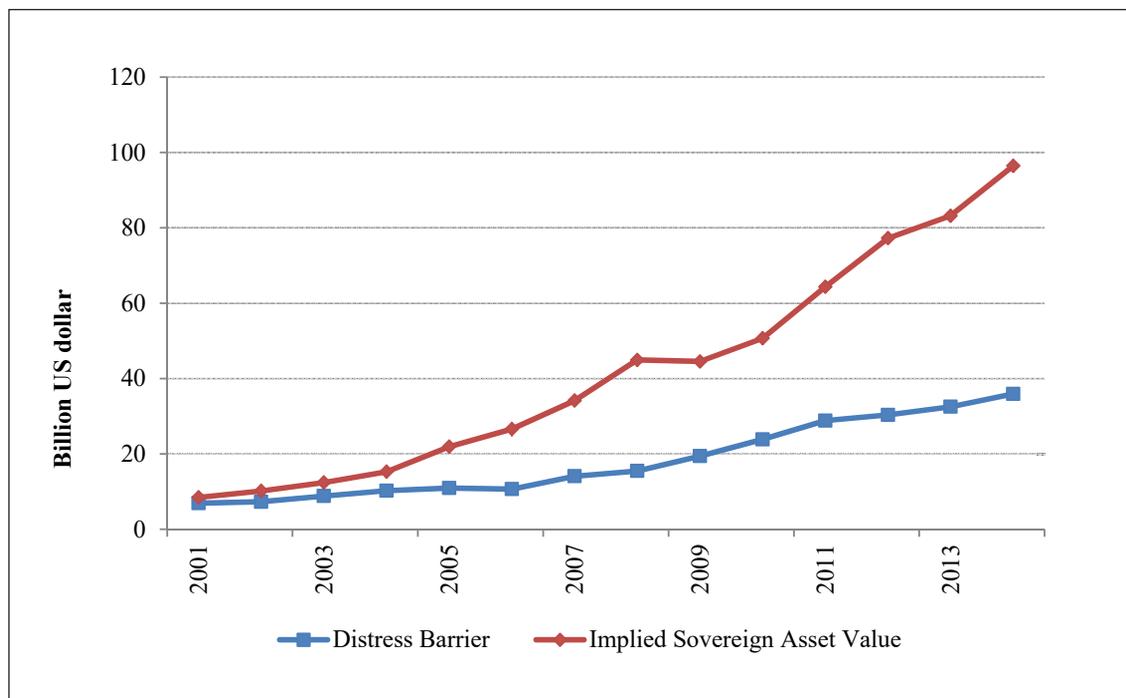
#### 4. Scenario analysis: Vietnam Government's pro-forma balance sheet

##### 4.1. The baseline

The starting point is the baseline balance sheet for the Vietnam government in 2015 as displayed in Table 2:

*Calibrated value:* The distress barrier is US\$50.16 billion; the value and volatility of

**Figure 4: Implied sovereign asset value versus distress barrier**



**Table 1: Correlation of sovereign risk indicators and sovereign bond spread**

Country	Distance-to-distress		Risk-neutral spread	
	Spearman's rho correlation	Sig. (2-tailed)	Pearson correlation	Sig. (2-tailed)
Indonesia	-0.767*	0.016	0.962**	0.00003
Malaysia	-0.436	0.08	0.753**	0.00048
Philippines	-0.676**	0.003	0.730**	0.00088
Vietnam	-0.818**	0.004	0.954**	0.00002

Notes: \*\*. Significance level of 1% (2-tailed).

\*. Significance level of 5% (2-tailed).

domestic currency liabilities in dollar terms are US\$305.6 billion and 0.38 (38 percent), respectively. Using equation (3) and (4), the implied value of sovereign assets is US\$349.66 billion and the implied volatility of sovereign assets is 0.33 (33 percent). Foreign currency reserves

make up US\$28.4 billion out of implied sovereign assets.

*Credit risk indicators:* From equation (5), the resulting distance-to-distress is 1.14 standard deviations. The distance-to-distress results in a risk-neutral default probability

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(RNDP) of 12.7% in equation (6). Equation (2) yields the value of risky foreign currency debt as US\$44.05 billion, implying a present value expected loss of US\$1.14 billion. This value is derived from the difference between the discounted present value of the distress barrier (using a risk-free rate of 0.65% yields a present value distress barrier of US\$46.3 billion) and the implied market value of foreign currency debt.

*Sensitivity measures:* Sensitivity measures are calculated from a 1 percent change in sovereign asset value and volatility. For example, when the value of sovereign assets decreases by 1 percent, Table 2 shows that the distance-to-distress falls by 0.01 standard deviations (i.e., from 1.14 to 1.13 standard deviations), risk-neutral default probability increases by 0.18 percent, risk-neutral credit spreads (RNS) increase by 0.67 basis points, and the expected loss on foreign currency debt increases by US\$37 million. Sensitivity measures are also reported for a 1 percent change in sovereign asset volatility.

#### **4.2. Scenario analysis**

Two scenarios for the Vietnam government in 2016 and 2018 are examined with the prediction from the International Monetary Fund (IMF) and the World Bank (WB). First, Vietnam's nominal gross domestic product (GDP) is projected to be US\$200.2 billion and US\$215.4 billion for 2017 and 2018, respectively. Second, for the two years of GDP growth, Vietnam's public debt is estimated to make up 64.4 percent of GDP in 2016 and reach the 65 percent ceiling rate set by the Ministry of Finance. Third, among the government's total debt, external debt is predicted to rise by US\$6 billion

to US\$56 billion in 2016 and reach US\$67.25 billion by 2018. The two right-most columns in Table 2 display the new contingent claim sovereign balance sheets, balance sheet risk indicators, and sensitivity measures under the impacts of economic growth and capital structure projection.

In the first scenario, since GDP and public debt ratio are both forecast to increase, the distress barrier rises to US\$55.9 billion in 2016 and that strengthens the repayment capacity of the Vietnam government. Distance-to-distress increases by 0.02 standard deviations (from 1.14 to 1.16) and risk-neutral default probability falls to 12.2 percent compared to 12.7 percent in the baseline. Risk-neutral spreads on foreign currency debt decrease to reflect the lower risk of non-repayment as the expected loss has fallen from 4.8 to 4.6 percent of the present value of the distress barrier. In addition to a strengthening of the credit risk indicators, the sensitivity measures have slightly decreased and as implied sovereign asset value is further from the distress barrier. In particular, a 1 percent decline in the sovereign asset value increases RNDP by 0.17 (from 12.2 to 12.39 percent) and RNS by 0.65 basis points, while these figures for the baseline were 0.18 and 0.67, respectively. The lower sensitivities reflect the lower degree of nonlinearity within the option pricing formula as sovereign assets move further away from the distress barrier, which resulted from a significant GDP growth rate and a negligible increase in public debt ratio for the Vietnam economy in 2016. In comparison, the distress barrier for the Vietnam government in 2018 is predicted to reach US\$67.3 billion and that worsens the credit risk indicators and risk

**Table 2: Forecast scenarios and contingent claim sovereign balance sheet risk indicators**

	<b>Baseline</b>	<b>2016</b>	<b>2018</b>
<b>Contingent Claim Sovereign Balance Sheet</b>			
	(US\$ billion, unless indicated)		
Value of sovereign assets (implied)	349.7	404.2	437.6
Foreign reserves (observed value) 1/	28.4	33.7	47.6
Sovereign asset less reserves (implied)	321.3	370.5	390.0
Value of risky foreign currency debt	44.1	49.2	58.8
Distress barrier 1/	50.2	55.9	67.3
PV of distress barrier	46.3	51.6	62.1
PV of expected losses (= implicit put option) 2/	4.8%	4.6%	5.2%
Value of local currency liabilities 1/	305.6	355.0	378.8
Volatility of assets (implied)	33.3%	33.4%	33.0%
<b>Credit Risk Indicators</b>			
Distance-to-distress 3/	1.14	1.16	1.10
Risk-neutral default probability (RNDP)	12.7%	12.2%	13.5%
Risk-neutral spread (RNS) 2/	40.1	38.5	43.1
<b>Sensitivity Measures<sup>5/</sup></b>			
Change in distance-to-distress / 1% change in assets 3/	-0.01	-0.01	-0.01
Change in distance-to-distress / 1% change in asset vol. 3/	-0.023	-0.023	-0.022
Change in RNDP / 1% change in assets	0.18%	0.17%	0.19%
Change in RNS / 1% change in assets 4/	0.67	0.65	0.72
Change in RNS / 1% change in asset vol. 4/	2.10	2.05	2.18
Change in PV expected loss / 1 % change in assets 2/	0.079%	0.077%	0.084%
Change in PV expected loss / 1 % change in asset vol. 2/	0.246%	0.241%	0.255%

1/ Model input for baseline

2/ In percentage of present value of distress barrier

3/ In standard deviation of sovereign asset value

4/ Spread in basis points

5/ Base on a 1 percent change in sovereign asset value (e.g. from 349.7 to 346.2) and sovereign asset volatility (e.g. from 33.3% to 33.63%)

exposure has escalated relative to the baseline. Distance-to-distress drops to just over one standard deviation and risk-neutral default probability increases by 0.8 percent. Risk-neutral spreads on foreign-currency debt and sensitivity measures both increase compared to the baseline as the value of risky foreign-currency debt approaches its default-free value.

Looking at the CCA indicators in detail, the main factor driving the risk exposure of the Vietnam government seems to be the volatility of assets that is implied in 2016 at 0.1% higher than the baseline. This figure practically means that the more volatile Vietnam sovereign assets get, the more significantly it can grow to cover the respective increase of the foreign curren-

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cy debt (11.4% from US\$50.2 to US\$55.9 billion). The deeper cause of the higher potential assets growth is attributed to the value of local currency liabilities comprising base money and local currency debt. In 2016, local currency liabilities increase by 16.2% which means the equity cushion for sovereign assets is thickened by roughly 5% higher than the rise of risk exposure (foreign currency debt). Note that the value of local currency liabilities is estimated in US dollars that can be affected by not only the base money and VND-denominated debt but also the volatility of the forward exchange rate. When the amount of money in circulation expands quickly exceeding the growth of GDP, which is indicated in the depreciation of domestic currency, the sovereign credit risk will continue unabated over time and the government will even have to confront additional inflation issues. This situation is predicted to emerge in Vietnam in 2018 as both base money and local currency debt grow by more than 8% but the forward exchange rate is projected at 28.93 thousand VN dong for one US dollar compared to only 26.3 thousand in 2016. As a result, a worsened credit risk is presented in the increase of risk-neutral probability (RNDP) and the higher possibility of change in RNDP for every 1% change in assets (0.19%) which all show a weaker ability-to-repay and that the Vietnam government will be more vulnerable against the changes in the value and volatility of its own assets.

## 5. Recommendations

The Sovereign CCA framework has a numerous applications for both investors and policymakers. The model provides a relatively valuable framework for contingent claims on

sovereign assets and can be a helpful tool for the analysis and management of a sovereign wealth fund (SWF). In addition, the CCA outputs on risk exposure form a concrete base for exploring new ways of transferring sovereign risk and developing potential new risk transfer contract arrangements. The development of such instruments is known as Alternative Risk Transfer (ART).

### 5.1. Application to sovereign asset and wealth management

The increasing popularity of sovereign wealth funds (SWF) over the last decade implies their important role in the global economy. The estimated assets value of US\$5.78 trillion (2013) plus US\$7.2 trillion of other sovereign investment vehicles such as pension reserve funds and US\$8.1 trillion in other official foreign exchange reserves is a substantial pool of funds totaling US\$20 trillion that many emerging economies, including Southeast Asia area, have access to.

Consider the four countries in question with different economies and different risk profiles. Indonesia is vulnerable to lower oil prices and higher fuels prices while Malaysia is at risk of lower prices of petroleum, palm oil and wood products. The Philippines is susceptible to higher raw materials, consumer goods and fuel prices and the uncertainty of capital inflow from the U.S, Japan and the EU. Finally, Vietnam is exposed to lower prices of marine and agriculture products, higher prices of petroleum, fertilizer, cotton and other intermediate goods. The four sovereigns have various exposures from tax revenues, expenditures, risks of banking system crisis and to capital inflows and outflows. Taking such risk profiles of the

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countries into account, the CCA framework allows the quantification of a “sovereign portfolio” consisting of reserves, fiscal and other assets including the contingent liabilities. This quantitative risk-oriented approach has two important advantages.

*First*, it is a potentially useful tool with which to gauge the risk reduction benefits of holding liquid foreign currency reserves versus other financial instruments for managing risks. Many SWFs are from Asian countries whose booming export sectors and commodity exporters have amassed large reserve positions. Reserves in excess of the required liquid reserves can be invested in higher return but less-liquid instruments. The CCA can be used to assess investment strategies that provide likely optimal hedging and diversification/risk reduction tailored for the risk characteristics specified for each country.

*Second*, the combination of the CCA and a Value-at-risk (VaR) type approach adjusted for a sovereign which is called as Sovereign Asset-at-Risk (SAaR) can evaluate investment strategies that keep the tail of the probability distribution of the sovereign asset portfolio above a threshold for a specific confidence level (e.g. 1%, 5% or 10%). If the sovereign debt structure of the country in question includes significant foreign-currency denominated debt, there may be additional debt targets. For example, the external debt of Vietnam is projected to exceed the target level of 65% GDP by 2018. As the threshold is broken, policymakers can adjust various components of the sovereign balance sheet to lower the risk, for example:

- Use fiscal, debt and other policies that change fiscal surplus, the amount and maturity

of outstanding government local currency and foreign-currency debt.

- Make changes in asset allocation with respect to the risks, volatility and covariance among the different components of the sovereign balance sheet.

- Use derivative securities to hedge the sovereign assets as described in detail in section 5.2.

Therefore, CCA provides a framework for assessing each economic sector’s assets and liabilities and allows policymakers to take a holistic view when formulating the asset mixture.

### ***5.2. Application of sovereign CCA framework for design of new risk transferring instruments and the “ART” of sovereign risk management***

The application of CCA to measure risk exposures in the economic sectors suggests a concrete framework for comparing alternative solutions to control and transfer risk. The field of ART includes a wide range of instruments and contracts used by firms, financial institutions, and insurance companies. The majority of these tools can be applied to directly or indirectly transfer sovereign risk.

Risk can be transferred by a change in the capital structure, by managing guarantees (i.e. policies to limit the obligations to too-important-to-fail entities), or through risk transfer. When the economy experiences distress, for example, the country balance sheet is likely to deteriorate, which consequently causes a drop in tax revenue and a spike in the cost of debt service for the government, resulting in a higher level of sovereign credit risk. Hence, these observations offer a powerful argument for di-

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versification of the sovereign exposure to local shocks. On a country level, diversification via international capital mobility is a popular solution. For a further step, the sovereign CCA suggests a variety of combined alternatives for diversification, hedging, or mitigation of sovereign risk.

- *Diversification and hedging through foreign reserves management* – CCA can be used for assessing pros and cons of increasing sovereign reserves via issue of external debt, compared to having an “equity cushion” to mitigate potential losses.

- *Sovereign bonds with special provisions* – Indexed bonds such as commodity or GDP-linked bonds can help manage risk in a way that an unprofitable economy results in lower contractual repayments on the government bonds.

- *Equity swaps as a method of diversification* – An equity swap would enable a small country to internationally transfer risk without violating restrictions on foreign investments. Such an instrument performs especially well for nations greatly dependent on specific exports. For example, Vietnam can reduce its dependence on agricultural commodities by involving in an equity swap whereby the Vietnamese government would pay returns on its agricultural commodities in exchange for returns on another industry, say the automobile industry. Thus, equity swap might be a solution for small countries like Vietnam to focus on the industries in

which they have comparative advantages and simultaneously retain efficient risk diversification.

## 6. Conclusion

The devastating results of international economic and financial crises over the last few decades entail an increasing importance for the analysis of the stabilization of the economy and the financial market. This paper adopts a modern financial analysis approach for the Vietnamese financial market. We find that the Contingent Claim Analysis (CCA) framework and the credit risk indicators for four Southeast Asia countries are helpful for assessing vulnerability, policy analysis, sovereign credit risk analysis and sovereign capital structure. Such a theoretical framework is also suitable for the design of sovereign risk mitigation and trading strategies. The scenario analysis is conducted with the projections on GDP growth and public debt figures from the World Bank and the IMF. From our findings, we derive recommendations for the application of CCA to Sovereign assets and wealth management as well as the *design of new risk transferring instruments and the “ART” of sovereign risk management*. The reassuring robustness checks of the CCA and the pro-forma sovereign balance sheet provides significant support to a Contingent Claim approach in measuring Vietnam’s sovereign risk.

## Appendix A: Markov property and Wiener stochastic process

In a continuous-time stochastic process, an important assumption under financial assets pricing theory is the Markov property, which assumes only the present value of variable is relevant for predicting the future, while the past history of the variable and the way that the present has emerged from the past are irrelevant. This means the probability distribution of the variable at any particular time is not dependent on the particular path followed by the price in the past. The Markov property is consistent with the weak form of market efficiency where the present price of a stock impounds all the information contained in a record of past prices, making above-average returns from technical analysis out of the ordinary.

The Wiener process is a particular type of Markov stochastic process, with a standardized normal distribution, mean change of zero, and variable rate of 1.0 per time unit. That means change and variable rate for a stochastic process are known as the “drift rate” and the “variance rate”, respectively. The basic Wiener process of a hypothetical variable,  $dz$ , has two properties:

1. The change  $\Delta z$  in a small time interval  $\Delta t$  is  $\Delta z = \epsilon\sqrt{\Delta t}$ , where  $\epsilon$  has standardized normal distribution  $\Phi(0,1)$ .
2. The values of  $\Delta z$  for any two different short period of time,  $\Delta t$ , are independent.

A variable,  $x$ , that follows a generalized Wiener process can be defined by terms of  $dz$  as

$$dx = adt + bdz, \text{ where } a \text{ and } b \text{ are constants}$$

The  $adt$  term implies the pace of growth in the value of  $x$  (e.g. return on an asset). The  $bdz$  term stands for additional noise or variability on the path followed by  $x$ . Combining the generalized equation with the property 1 of a basic Wiener process, we obtained

$$\Delta x = a\Delta t + b\epsilon\sqrt{\Delta t}$$

Turning to our corporate asset return, the generalized Wiener process for an asset value,  $A$ , is defined as

$$dA = \mu_A A dt + \sigma_A A \epsilon \sqrt{t}, \text{ or}$$

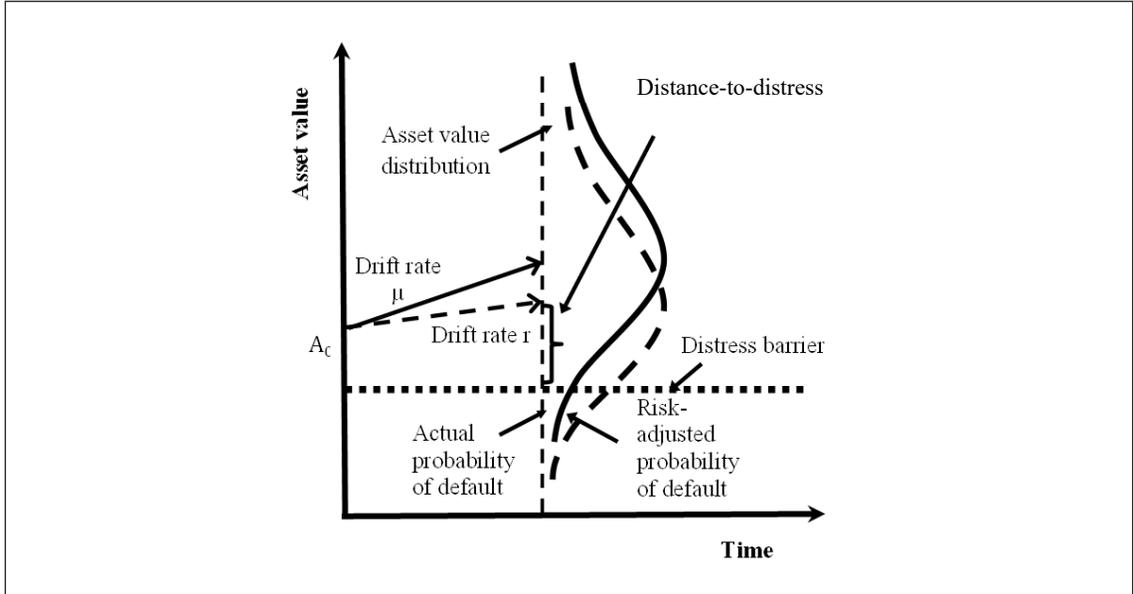
$$\frac{dA}{A} = \mu_A dt + \sigma_A \epsilon \sqrt{t},$$

where  $\mu_A$  is the drift rate,  $\sigma_A$  is standard deviation of the asset return, and  $\epsilon$  is normally distributed with zero mean and unit variance.

The probability distribution of asset value at time  $T$  is illustrated as the solid line in Figure 1. Default occurs when the asset value falls to or below the promised payment,  $B_t$ , that is also called the “distress barrier”. The probability of default is the probability that  $A_t \leq B_t$  which is  $\text{Prob}(A_0 e^{(\mu_A - \sigma_A^2/2)t + \sigma_A \epsilon \sqrt{t}} \leq B_t) = \text{Prob}(\epsilon \leq -d_{2,\mu})$ . Since  $\epsilon \sim N(0,1)$ , the actual probability of default is  $N(-d_{2,\mu})$ , where  $d_{2,\mu} = \frac{\ln(A_0/B_t) + (\mu_A - \sigma_A^2/2)t}{\sigma_A \sqrt{t}}$ .  $N(\bullet)$  is the cumulative standard normal distribution.

The asset return probability distribution used in contingent claims analysis is not “actual”, but the “risk-adjusted” distribution, which replaces the risk-free interest rate,  $r$ , for the actual expected return,  $\mu_A$ , in the calculation. This is called risk-neutral probability distribution and is plotted as the dashed line in Figure 1. Thus, the risk-adjusted probability of default is larger than the actual one for all assets with an actual expected return greater than the risk-free rate, which rationally requires a positive risk premium. The formula for risk-adjusted probability of default is  $N(-d_2)$  where  $d_2 = \frac{\ln(A_0/B_t) + (r - \sigma_A^2/2)t}{\sigma_A \sqrt{t}}$ . The risk-free rate is used due to the assumption about a risk-neutral world in the Black-Scholes option-pricing model which the CCA was based on. Calculating the actual probability of default is outside the Merton’s model, but it can be combined with an equilibrium model of underlying asset return to derive consistent estimates for expected return on all derivatives. This drawback is caused by simplifying the model with no requirement on estimating the assets expected return for the purpose of value or risk measures.

**Figure 5: Probability distribution of asset value in relation to Distress barrier**



### Appendix B: Pearson's and Spearman's correlations

Correlation coefficient 'r' is a number that represents the level of relationship between two individual variables (Washington et al, 2010). In statistics, there are two main measurements of correlations: Pearson product-moment correlation (Pearson's correlation) and Spearman rank-order correlation (Spearman's rho correlation). The Pearson's correlation coefficient applied to a sample is commonly represented by the letter  $r$ . If we have one dataset  $\{x_1, \dots, x_n\}$  containing  $n$  values and another dataset  $\{y_1, \dots, y_n\}$  containing  $n$  values then that formula for  $r$  is:

$$r = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2} \sqrt{\sum_{i=1}^n (y_i - \bar{y})^2}}$$

where  $\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$  is the sample mean; and analogously for  $\bar{y}$ .

The Spearman's correlation coefficient is defined as the Pearson's correlation coefficient between the ranked variables and is denoted by letter  $r_s$ . For a sample of size  $n$ , the  $n$  raw scores  $X_i, Y_i$  are converted to ranks  $rgX_i, rgY_i$ , and  $r_s$  is computed as:

$$r_s = \rho_{rgX, rgY} = \frac{cov(rgX, rgY)}{\sigma_{rgX} \sigma_{rgY}}$$

where  $\rho$  is the usual Pearson correlation coefficient, but applied to the rank variables

$cov(rgX, rgY)$  is the covariance of the rank variables

$\sigma_{rgX}$  and  $\sigma_{rgY}$  are the standard deviations of the rank variables.

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## Notes:

1. The contingent claims approach was applied to estimate balance sheet risk in the aggregated corporate sector in Gapen, Gray, Lim, and Xiao (2004). The analysis also provided estimates of risk transfer across the corporate, financial, and public sectors.
2. We are considering only “European” options, in which the option can be exercised only at maturity, as opposed to “American” options, in which the option can be exercised at any time. In our application, this means that a firm can go bankrupt only at debt’s maturity.
3. Risk-neutral world is an important assumption underlying the derivation of the Black-Scholes option pricing formula whereby the value of the option can be derived by forming a riskless hedge portfolio. Thus, option values do not depend on the investor’s attitude toward risk, which is a major benefit of this approach. See Chriss (1997, pp.190–193) and Hull (2012, pp. 280–329) and for additional discussion of risk-neutral valuation.
4. Eichengreen et al. (2002) support the view of foreign currency debt as senior, and Sims (1999) argued that that local currency debt has many similarities to equity issued by firms. Sims modeled domestic currency debt as “equity”, considering domestic currency debt as a cushion and risk absorber of fiscal risk for a sovereign.
5. The implicit guarantees to the financial sector, or other entities, could remain on the liability side of the consolidated public sector balance sheet and modeled as implicit put options. More details can be seen in Merton (1977); Gray et al. (2002); Gapen, et al. (2005); and Van den End and Tabbae (2005). These papers link the sovereign to the contingent claim balance sheets of the banking or corporate sectors.
6. This definition of the distress barrier is similar to that used by Moody’s KMV in corporate sector default risk analysis (Crosbie and Bohn, 2003). Short-term is defined as one year or less by residual maturity.
7. It should be noted that this ordering can be flexible and the contingent claims framework can be adapted to any number of different seniority structures. In future work, the seniority assumption will be relaxed to take into account multiple layers of liabilities.

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